Chapter 4 Valuing Bonds

Chapter Overview

The *What Companies Do* opening feature discusses the impact the line from the film *Forrest Gump* had on Hotel Chocolat. Instead of paying dividends on a large necessary bond issue – they paid in chocolate. During the credit crunch, Hotel Chocolat issued bonds to their Tasting Club. The club would receive dividends in chocolate worth the amount of the dividend payment and then receive cash at the end of the issue. What a sweet way to pay dividends!

***What Companies Do* Discussion Questions**:

1. Why would an investor be willing to receive a dividend in chocolate? Why or why not? Why was this unique bond issue successful? Relate this to the relevant cash flows for such a security.
2. Under what circumstances would there be a financial interest in an issuer refinancing a bond?

This chapter covers important valuation models for bonds. Later chapters will go into more detail about the securities, while this chapter presents the concept that each security has a set of future cash flows and an appropriate discount rate, and with this information, a value can be calculated.

This chapter discusses:

4-1 Valuation Basics
4-2 Bond Prices and Interest Rates
4-3 Types of Bonds
4-4 Bond Markets
4-5 The Term Structure of Interest Rates

Technology

1. **Smart Ideas.** Todd Richter, MD, Head of Equity Healthcare Research, Bank of America Securities. 'The concepts of value, the things that *drive* value, don't change.
2. **Smart Ideas.** Annette Poulsen, University of Georgia. 'There is a trade-off between flexibility for the corporation and protection for the bondholder.'
3. **Smart Concepts.** See how to apply time value of money concepts to value shares and bonds. See a demonstration of the expectations theory of term structure.
4. **Smart Solutions.** Step-by-step solutions to Problems 4-8 and 4-20.

After completing this chapter you should be able to:

- recall the fundamental concepts that determine how to value assets
- understand the vocabulary that describes bonds and the markets in which they trade
- interpret the relationship between bond prices and interest rates
- explain the meaning of the term 'structure of interest rates'.

Lecture Guide

This is a key chapter, introducing bond and share valuation models. Not only do individual investors want to know how to value these securities, but corporate managers need to know about valuation. A company may be in the market for debt or equity financing, and a manager who wants to maximise share price also needs to know the valuation models and factors that affect share price.

4-1 Valuation Basics

The instructor should point out the recurring theme throughout this textbook, that there is a link between risk and return. The greater the risk, the greater the required return and the greater the discount rate used on the company's cash flows, and the lower the valuation. This is a fundamental principle of finance and should be emphasised.

4-1a The Fundamental Valuation Model

The fundamental valuation model takes the time value of money techniques and applies them to the cash flows of a company, security, capital investment, etc. The 'i' or interest rate in the previous chapter is now 'r' the opportunity cost or required return on an investment. The model itself is very simple: just take the series of cash flows that the asset is expected to provide and discount each cash flow back to today's dollars. The P represents the asset's price today and the CF represents the cash flow at time t. The equation will provide the price that reflects the assets risk.

4-2 Bond Prices and Interest Rates

4-2a Bond Vocabulary

Make note of bond terminology for students and the fact that different terms can be used to describe the same feature. For example, the par value of a bond is also referred to as its face value or face amount of principal value. Typically a bond is worth its face value only at its issue and at its maturity. A good way to show the students a practical example is to have them open the bond section of any financial website (Yahoo Finance or Bloomberg) and identify the parts, or terms, of the bonds available for any company.

4-2b The Basic Equation (Assuming Annual Interest)

In a bond problem, you could be given either the coupon interest rate (the per cent yearly interest paid by the bond), or you could be given the dollar amount of interest paid by the bond. If you are given a coupon interest per cent, you can compute the dollar interest by multiplying the face value of the bond by the coupon interest rate. This represents the C in the equation and is the cash flow the bondholder receives each predetermined pay period.

The interest rate on a bond depends on prevailing market conditions at the time of issue – what is the cost based on the length of time of the bond and the risk of the company issuing the bond. This represents the opportunity cost of capital, what could be earned on other investments of the same risk.

Figure 4.1 Time Line for Bond Valuation (Assuming Annual Interest Payments)

Yield-to-Maturity

The yield-to-maturity of a bond is its internal rate of return – the returns investors who hold the bond to maturity will get. The return depends on the amount and timing of cash flows from the bond.

Note that yield to maturity is like the interest rate used in the time value of money calculations in the previous chapter. Note also that yield to maturity is determined after bond prices – what the market will pay – are known. Yield to maturity is what an investor would earn if he bought the bond and held it until it matured. It is difficult to calculate yield to maturity without a financial calculator. Using the formula means solving for r using trial and error, a very time-consuming process.

An approximation formula for yield-to-maturity gives a reasonably close approximation of yield to maturity for low yields. It is not as accurate for very high-yielding bonds. The formula states:

$$YTM = \frac{I + (F-P)/N}{(F+P)/2}$$

I = dollar amount of interest
F = face value of bond
P = price of bond
N = number of years to maturity

Suppose a $1,000 face value bond is selling for $1,100. The bond has 10 years left to maturity. The bond has an 8% coupon rate. Using the approximation formula, the YTM is:

$$\frac{80 + (1000 - 1100)/10}{(1000 + 1100)/2} = 70/1050 = 6.7\%$$

Solving for YTM using a financial calculator, the solution is:
N = 10
PMT = 80
FV = 1,000
PV = –1,100
Solve for I = 6.6%

Now suppose you have a deeply discounted bond, which could result if a company were financially distressed and investors were reluctant to buy the bonds for fear that they would not receive future payments. A $1,000 face value bond is selling for only $200. The bond has 10 years to maturity and a coupon payment of $80. Using the approximation formula, the YTM is:

$$\frac{80 + (1000 - 200)/10}{(1000 + 200)/2} = 160/600 = 26.7\%$$

Calculating YTM using a financial calculator yields:
N = 10
PMT = 80
FV = 1,000
PV = –200
Solve for I = 44.6%

However, it is very rare for a bond to have such a high yield; this would only occur with a very deeply discounted bond. Note also that this is a way to compute IRR without using a financial calculator.

Bond Premiums and Discounts

Understanding yields is an important part of bond valuation. Bond prices are most impacted by changes in the interest rate – if interest rates increase, bond prices decrease and if interest rates decrease, bond prices increase. Sometimes this is a difficult concept for students to understand because it would intuitively seem that higher interest rates should make more bonds more valuable. This is true for when you are comparing two identical bonds with differing interest rates. If two bonds have the same risk and maturity characteristics, and one carries a coupon rate of 10% and the other a coupon rate of 12%, the 12% bond will be more valuable, and will sell for a higher price.

Now suppose you own the 10% bond. A year later, interest rates increase to 12%. Now a new investor would have the choice of investing in your bond with a coupon rate of 10% or a new bond with a coupon of 12%. Your bond has become less valuable, compared to what an investor could purchase in the current market with the same risk. So, if you wish to sell your bond, you will have to sell it at a discount, at less than its full face value. Similarly, if interest rates decline to 8% after you purchase your 10% bonds, your bond is now more valuable. Investors would want a security paying 10% coupon rate when the going rate is only 8%. Now, your bond will sell at a premium, at more than face value.

4-2c Semiannual Compounding

In bond calculations, note that price (PV) must be negative while coupon payment (PMT) and face value (FV) are positive. Mathematically, the inputs in a calculator would still give you the same answer if you inputted a positive number for price and negative numbers for face value coupon payment. However, it is generally easier to think of a bond's price as negative because that is what you must pay to get the value of future cash flows.

Many government and Australian company bonds pay on a semi-annual basis. To calculate bond value on a financial calculator if the payment is semi-annual, the number of years is multiplied by two, the interest rate is divided in half and the payment is divided in half. The price (PV) and face value (FV) remain the same and are not changed in semi-annual bond calculations. One common error with semi-annual bond calculations using a financial calculator is that students who are solving for I (yield to maturity) may forget to multiply their answer by two to get an annual yield to maturity.

It is also helpful to bring in the equivalent annual return concept from the previous chapter. If a bond's stated coupon rate is 10%, an investor is really receiving $(1.05)^2 = 0.1025$ or 10.25% annually. The semi-annual payments received increase the interest rate (which is usually stated as an annual rate).

4-2d Bond Prices and Interest Rates

Bonds may be premium or discount bonds at various times after the bond is issued. However, as the bond approaches maturity, it will once again converge toward the par value it had at issue.

Note that longer-term bonds are more sensitive to changes in the interest rate than are shorter-term bonds, and that lower-coupon bonds are more sensitive to interest rate changes than are higher coupon bonds. The former is true because a changed discount rate affects cash flows over a longer period of time in a longer-term bond. When the coupon rate is lower, a given change in interest rates represents a larger percentage change in the current coupon rate.

A bond is the sum of the discounted value of the coupon interest payments plus the discounted principal repayment. Note also that as a bond approaches maturity, the discounted value of the principal amount becomes a larger and larger proportion of the bond's value.

Figure 4-2 The Relationship Between Bond Prices and Required Returns for Bonds with Differing Times to Maturity but the Same 6% Coupon Rate

Interest Rate Risks: (Including Inflation and Issuer Risk)

Price fluctuations will not affect an investor who wishes to hold a bond to maturity. However, an investor should not simply hold a bond to maturity to avoid loss of principal. Investors should be looking at alternative investments that could offer higher returns. It could be worthwhile to sell a bond when it is high priced if it is possible to reinvest at an even higher rate.

A bond's risk includes the risk that interest rates will change and reinvestment rate risk, the risk of reinvesting your intermediate cash flows (coupon payments) and principal repayment. A short-term bond carries more reinvestment rate risk for the investor – the investor will receive interest and principal cash flows and must decide how to reinvest those funds. A long-term bond carries more interest rate risk, the risk that interest rates will change after the bond is purchased and adversely impact the bond's price.

Ask students which kind of risk is more important, interest rate risk or reinvestment rate risk? For a long term or short-term investor? Most students will agree that long-term investors face more risk, and as such will require a higher return for long-term bonds. This is a particularly difficult problem for retirees. They may prefer to invest long-term so they lock in an interest rate for a longer period of time. On the other hand, investing for the long term leaves an investor more susceptible to inflation. High inflation could erode the purchasing power of their long-term investments. This leads into a discussion of yield curves. Most of the time we have an upward sloping yield curve where long-term interest rates are higher than short-term rates.

Figure 4.3 Australian Government Bond Yields 1975-2012

Figure 4.4 Australian Inflation Rate

4.3 Types of Bonds

Bonds have primary and secondary markets, just as shares do. However, in general bonds are less liquid than shares. Anyone wanting to buy or sell BHP Billiton shares will find a wider range of brokers – both online and in conveniently located offices – than someone wanting to buy or sell BHP Billiton bonds. Also, bond sales generally involve larger amounts of money.

4-3a By Issuer

Corporate bonds are issued by large companies who need money to finance new investments. Most Australian company bonds have a par value of $100 and pay interest semi-annually. The Australian company bond market was very small up to 2000, but has grown rapidly since then.

4-3b By Features

This section can also be related to the risk-return relationship. Which is riskier, floating rate or fixed rate debt? The answer depends on the point of view you take. Floating rate debt is less risky to the investor and more risky to the company issuing the debt. Investors are ensured that if interest rates rise, their bonds will not decrease in value. Companies, on the other hand, will have higher expenses if interest rates rise. Floating rate bonds would be expected to have lower coupon interest rates than equivalent fixed rate bonds.

Similarly, look at the risk differences of secured vs unsecured bonds. Secured bonds are less risky for the investor and will carry a lower coupon rate than unsecured bonds. Secured bondholders will be able to stake a primary claim on the assets used to secure the bonds and will be more likely to be paid back in insolvency than unsecured creditors.

Note that zero coupon bonds are highly discounted because they have no coupon payments, only a return of principal at maturity. Convertible and exchangeable bonds are part share and part bonds, and are valued as such. Valuing convertible bonds is beyond the scope of this chapter and requires option pricing theory – a convertible bond contains an option to exchange a debt security for an equity security.

Many bonds have a call feature. An example of a US company that had bonds with no call features and might have wished for one is Burlington Northern Railroad. In the late 1890s Burlington issued 100 and 150 year bond issues. At the time, railroads were king and were given large land grants on which to build the railroads. The bonds had a provision requiring all money from the land to be re-invested in the railroad business. Fast forward to the late 1980s – now the railroad business has declined and further investment is not worthwhile. The land, however, including mineral, timber and development rights, is very valuable. Burlington was stuck. It wanted to exploit the value of the land, but it did not want to reinvest the proceeds into a declining business. Bondholders knew that they had an advantage and required Burlington to pay them a cash settlement to retire the bonds. If the bonds had a call feature, this might not have been necessary. Again note the risk return relationship. A call feature is advantageous to the company and means a callable bond should carry a slightly higher interest rate than an equivalent non-callable bond. Bonds with covenants that protect bondholders similarly will carry slightly lower coupon rates than bonds that are not as highly protected. For example, Marriott Corporation divided itself into two companies, Marriott International and Host Marriott. Marriott International became a hotel management company and Host Marriott primarily a real estate and food service company. Most of Marriott Corporation's debt was transferred to Host Marriott and Marriott International shares paid to shareholders as a special dividend. Marriott was able to do this because Marriott's bonds were weakly protected. They did not contain a common covenant prohibiting the use of assets to pay dividends. Since Marriott's bonds were subject to more event risk, they would be expected to carry a higher coupon rate than more protected bonds.

Table 4.1 Zero-Coupon Bond Prices and Taxable Income

4-4 Bond Markets

The global bond market is much larger than the global stock market in terms of dollar volume of securities each day. Particularly since the global financial crisis of 2007-2009, many governments have

greatly increased their issuance of bonds. The Australian Government moved from a surplus to a deficit on its annual budget through the financial crisis, and financed the large deficits with bond issues.

4-4a Bond Price Quotations
Bond prices, like share prices have different bid and ask prices. The difference, or spread, is the transaction cost of making the trade. Bond traders take a fee for bringing together buyers and sellers.

Table 4.2 Retail Bond Issue by Heritage Bank, 6 October 2012

Corporate Bond Quotations

In Table 4.2, or any bond listing, note that bond prices are quoted as if a $100 bond were being traded. This will be the case even if the bond face value is $1000 or some other value. This means that the price is a per cent of par value. It is very easy to compute the market value of a bond issue if you are given the price. Turn the price into a per cent. As an example, suppose the last price is 107.161%. Multiply the price times the face value of the bond to get the bond value. For example if you were buying a $10,000 bond, the price would be 101.819% x 10,000 = $10,181.90. This bond is a premium bond if the prevailing market interest rate for bonds with these characteristics is less than the coupon rate of the bond.

4-4b Bond Ratings
Note that some bonds are junk ('high yield') bonds at issue, when a poorly rated entity issues the bonds. Some investment grade bonds become junk bonds, called fallen angels, if the company's credit rating drops below investment grade over the life of the bond. When Marriott Corporation announced its proposed split, its bonds dropped from an investment grade BBB to junk bonds. The bond prices dropped, with some issues losing as much as 30% of their value. Investors realised that without all of the assets and cash flows of Marriott Corporation, Host Marriott's bonds would become much riskier, with a greater chance of default. (Note that Host Marriott actually did well after the split, with its equity increasing in value and no difficulty meeting its debt obligations.) Rating can be difficult to interpret and it is always good to look and see why a bond has been rated the way it has been.

Table 4.3 Bond Ratings

Table 4.4 The Relationship Between Bond Ratings and Spreads at Different Maturities in the US at a Given Point in Time, Expressed in Basis Points

4-5 The Term Structure of Interest Rates
Note that term structure is not the future rate of interest – it is the rate that is in effect today for bonds of differing maturities.

4-5a The Yield Curve
Bond yields vary with maturity. The relationship between time to maturity and yield to maturity for bonds of equal risk is referred to as the term structure of interest rates.

Figure 4-5 Yield Curves for Australian Government Bonds

For a large bond market, the US is a great example. The website www.smartmoney.com/bonds/ has a living yield curve, which shows how today's term structure compares to past and average term structures. This web site also gives examples of flat and inverted yield curves and the economic conditions at the time of these less common shapes for the yield curve. It is very impressive to students to show this in class, if the classroom has Internet access.

4-5b Using the Yield Curve to Forecast Interest Rates

Figure 4-6 The Expectations Theory

4-5c The Liquidity Preference and Preferred Habitat Theories

4-5d Conclusion

Note that like any asset bonds are the discounted sum of their cash flows. The risk-return relationship is very important. Coupon rate reflect the risk of the company and reflect the features of the bond – whether they are advantageous to the company or to the investor. While interest rates are the primary factor influencing bond prices, the financial situation of the company may also be a factor. In other words, if a company becomes financially distressed, its bonds will react to changes in good and bad information released about the company, even more than to changes in interest rates.

Chapter 4 Resource Articles

'Widows and Orphans Beware: Treasuries Might Not Be So Safe', *Wall Street Journal*, 18 September 2002. As the stock market declined, many investors turned to bonds. As bond prices rise, yields are driven down. The yield on the benchmark 10-year Treasury note is less than 4%, the lowest it has been since the 1960s. Treasuries are risky because there is a floor to interest rates (0%) but much more room to rise. As yields fall, bonds become more volatile. Also, the spread between Treasury and corporate bond yields is unusually wide.

'Investors Love Their Bond Funds – Too Much?' *Wall Street Journal*, 9 November 2001. This article notes the increase in investment into bond mutual funds. If interest rates go up, bond prices will go down. Bonds are not necessarily the safe harbor investors are looking for.

Enrichment Exercises

1. Quiz students with easy (no calculation) concept questions. For example, suppose you wish to value a 10 year bond, with a face value of $1,000 and a coupon rate of 10%: r is equal to 10%. What is the value of the bond? While you can show students the calculations, point out that when the discount rate equals the coupon rate, then the bond's face value is its current price. No calculations need to be made.

2. Demonstrating the changes in yield curve over time is very impressive in a classroom. If you have internet access in your classroom, go to http://www.smartmoney.com/bonds and click on Living Yield Curve. If you cannot do this in the classroom, ask students to go to this web site on their own and compare today's yield curve with past yield curves. Could they have used the shape of the yield curve to predict the 2008-2010 downturn in the US stock market? What was the yield curve like in 2000 at the peak of the stock market? What changes have taken place since then?

Answers to Concept Review Questions

1. Managers need to understand how bonds and shares are priced because (1) companies regularly issue shares and bonds to raise money for investment, (2) understanding how securities are priced is helpful when conducting an acquisition or a divestiture, (3) the share price is an objective signal of how managers are performing, and (4) finance theory teaches that the goal of the manager should be to maximise the company's share price.

2. Holding future cash flows constant, the asset's price falls if risk rises because those future cash flows will be discounted at a higher rate.

3. Holding risk constant, an increase in expected future cash flows will increase the asset's price today.

4. According to Equation 4.1, the price of the land would depend on the cash flow generated by growing and selling crops. The price would depend on the crop yield, i.e., how much of a given type

of crop could be harvested in one acre, the selling price of the crop, and the costs of producing the crop.

5. The coupon rate equals the annual coupon payment divided by par value. The coupon yield equals the annual coupon payment divided by the bond's market price.

6. A bond sells at a discount when the bond's coupon rate is lower than the market's required rate of return on the bond.

7. 'Interest rate risk' refers to the possibility that a bond's price will change because the market's required return on that bond changes. From the perspective of the bond investor, such risk is especially of concern when their bond's price falls in response to an increase in the required return of the bond.

8. The cash flows (bond yields) of ordinary bonds are contractually fixed. Thus, an increase in bond yields means that these cash flows are being discounted at a higher rate, resulting in a lower present value or price of the underlying asset – in this case, a bond. The opposite is true if bond yields fall.

9. The Australian and New Zealand national governments, state and local governments in Australia, and companies are the main bond issuers in Australia and New Zealand.

10. A pure discount bond makes no coupon payments, while an ordinary bond selling at a discount makes coupon payments that are below the market's required return. The difference between the bond's discounted price and its value at maturity makes up the difference between the lower coupon payments and the market's required return.

11. The option to convert bonds into ordinary shares benefits bondholders. Once the share price rises high enough, the value of the bonds starts to behave like the share's value – the prices start to rise. So convertible bonds offer investors some minimal level of return plus a lot of upside potential. An issuer benefits from an option to call a bond, because such an option allows the issuer to lock in a more favourable interest rate if rates should fall.

12. The YTM is the rate that solves the standard bond pricing formula when given the current price, coupon rate, par value and maturity date of the bond.

13. The dollar price of a corporate bond quoted as 97.84%, with a $1,000 par value, is $978.40.

14. The yield spread on corporate bonds versus government bonds must always be positive due to risk. Corporate bonds are riskier, so they must offer higher yields than government bonds do. This means the yield spread is always positive. As a bond becomes more favourably rated (higher quality) it is less risky resulting in lower yields. Longer term bonds are subject to more interest rate risk than shorter term bonds, holding bond rating constant, and thus have higher yields.

15. The height of the yield curve depends on inflation because investors are aware that inflation erodes the value of a fixed cash payment. As a result, when investors expect high inflation, they will require high returns on bonds. When inflation expectations are lower, investors will accept lower rates on bonds.

16. Inflation-indexed bonds provide protection against inflation. Because the inflation-indexed coupon payments rise with inflation, the coupon rate on them is effectively fixed in real terms rather than in nominal terms. An ordinary bond offers a fixed nominal coupon rate, and this rate must be set high enough to convince investors that it also compensates them for inflation. Therefore, the coupon rate on an ordinary bond, which is expressed in nominal terms, must be higher than the coupon rate on

an inflation-indexed bond, which is expressed in real terms. The inflation-indexed bond will have coupon rate of less than 4%.

Answers to Self-Test Problems

ST4-1. A 5-year bond pays interest annually. Its par value is $1,000 and its coupon rate equals 7%. If the market's required return on the bond is 8 per cent, what is the bond's market price?

A: $$P = \frac{\$70}{1.08^1} + \frac{\$70}{1.08^2} + \frac{\$70}{1.08^3} + \frac{\$70}{1.08^4} + \frac{\$1{,}070}{1.08^5} = \$960.07$$

You could also obtain this answer by valuing the annuity of coupon payments and the lump sum principal amount separately as follows.

$$P_0 = \$70\left[\frac{1 - \frac{1}{(1+0.08)^5}}{0.08}\right] + \frac{\$1{,}000}{(1+0.08)^5}$$

$$= \$279.49 + \$680.58 = \$960.07$$

ST4-2. A bond that matures in 2 years makes semiannual interest payments. Its par value is $1,000, its coupon rate equals 4%, and the bond's market price is $1,019.27. What is the bond's yield to maturity?

A: The YTM is the value of r that solves this equation.

$$\$1{,}019.27 = \frac{\$20}{(1+\frac{r}{2})^1} + \frac{\$20}{(1+\frac{r}{2})^2} + \frac{\$20}{(1+\frac{r}{2})^3} + \frac{\$1{,}020}{(1+\frac{r}{2})^4}$$

Because the bond sells at a premium, the YTM must be less than the coupon rate. We can try to find the YTM by trial and error. Inserting r = 0.035 into the equation produces a price of $1,009.58. This price is too low, so we have chosen a YTM that is too high. Next try r = 0.03. At that interest rate, the market price is $1,019.27, so the YTM = 3%.

An alternative approach to this problem uses the Excel function, =IRR. This function requires that you input the price of the bond as a negative value, followed by the positive cash flows that the bond promises.

	A
1	–1,019.27
2	20
3	20
4	20
5	20

Now in an empty cell type the function, = IRR(A1:A5), and Excel will return the value 1.5%. This is the YTM stated on a semiannual basis (equivalent to r/2 in the equation above), so multiply it times 2 to get the annual YTM of 3%. Note, you need to be sure that the cell in which you type the IRR formula is formatted in a way that allows you to see several decimal places. Otherwise, Excel may round off the YTM and you will not know it.

ST4-3. Two bonds offer a 5% coupon rate, paid annually, and sell at par ($1,000). One bond matures in two years and the other matures in ten years.
a. What are the YTMs on each bond?
b. If the YTM changes to 4%, what happens to the price of each bond?
c. What happens if the YTM changes to 6%?

A: Because the bonds currently sell at par, the coupon rate and the YTM must be equal at 5%. If the YTM drops to 4%, both bonds will sell at a premium, but the price of the ten-year bond will increase more than the price of the two-year bond.

$$P_{2-yr} = \$50\left[\frac{1-\frac{1}{(1+0.04)^2}}{0.04}\right] + \frac{\$1{,}000}{(1+0.04)^2}$$

$$= \$94.30 + \$924.56 = \$1{,}018.86$$

$$P_{10-yr} = \$50\left[\frac{1-\frac{1}{(1+0.04)^{10}}}{0.04}\right] + \frac{\$1{,}000}{(1+0.04)^{10}}$$

$$= \$405.55 + \$675.56 = \$1{,}081.11$$

Repeating the calculations above at r = 0.06 we find that the two-year bond's price falls to $981.67 and the ten-year bond's price falls to $926.40. This illustrates that long-term bond prices are more sensitive to changes in interest rates than are short-term bond prices.

ST4-4. The nominal rate of interest is 5% and the expected inflation rate is 2%.
a. What is the approximate real rate of return? What is the exact real rate?
b. Repeat part (a) but assume that you are in a country experiencing hyperinflation, so the nominal interest rate is 90% and the expected inflation rate is 80%.

A. a. The approximate real rate is 3% (5% – 2%) and the actual real rate is 2.941% ([1 + 5%] / [1 + 2%] – 1).
b. The approximate real rate is 10% (90% – 80%) and the actual real rate is 5.556% ([1 + 90%] / [1 + 80%] – 1)…notice how the approximation becomes poorer given these rates.

Answers to End-of-Chapter Questions

Q4-1. What is the relationship between the price of a financial asset and the return that investors require on that asset, holding other factors constant?

A4-1. Holding an asset's cash flows constant, if investors pay a higher price for the asset, then their return from holding the asset will be lower. In general, asset prices are inversely correlated with returns.

Q4-2. Define the following terms commonly used in bond valuation: (a) *par value*, (b) *maturity date*, (c) *coupon*, (d) *coupon rate*, (e) *coupon yield*, (f) *yield to maturity* (*YTM*), and (g) *yield curve*.

A4-2. The par value is the face value of principal amount that a bond repays when it matures. It is usually $1,000 for corporate bonds. The maturity date indicates when a bond's final payment is due, and it signals the end of the bond's life. The coupon is the dollar amount of interest that a bond pays over a year. The coupon rate equals the coupon divided by the par value. The

coupon yield equals the coupon dividend by the market price of the bond. The YTM is the discount rate that equates the present value of a bond's cash flows to its current market price, and it is a measure of the return that investors require on a particular bond. The yield curve is a graph showing how interest rates vary with maturity for a group of bonds having equal risk.

Q4-3. Under what circumstances will a bond's *coupon rate* exceed its *coupon yield*? Explain in economic terms why this occurs.

A4-3. This occurs when the bond sells at a premium. This can occur when interest rates decrease making existing bonds more valuable and potentially selling at a premium.

Q4-4. What is the difference between a *pure discount bond* and a bond that trades at a discount? If issuers successfully sell pure discount bonds in the market, investors must want them. Can you explain why any bond purchaser might prefer to purchase a pure discount bond rather than a bond that pays interest?

A4-4. A pure discount bond pays no interest. A bond that sells at a discount pays interest at a rate that is below the market's required rate of return. The advantage of buying pure discount bonds is that these types of bonds always sell below par and a capital gain only needs to be recognised at maturity if holding the bond to maturity.

Q4-5. A company issues a bond at *par value*. Shortly thereafter, interest rates fall. If you calculate the *coupon rate*, *coupon yield*, and *yield to maturity* for this bond after the decline in interest rates, which of the three values is highest and which is lowest? Explain.

A4-5. As rates fall the bond's price will rise. This does not affect the coupon rate, but it will lower the coupon yield and the YTM. Because the bond now sells at a premium, there is a built-in capital loss for the investor who paid par value and holds the bond to maturity. Compensating for this loss is the fact that the bond's coupon yield will be greater than the YTM. So in order we have:

Coupon rate > coupon yield > YTM

Q4-6. Twenty-five years ago, the Singapore government issued thirty-year bonds with a *coupon rate* of about 8%. Five years ago, the Singapore government sold ten-year bonds with a *coupon rate* of about 5%. Suppose that the current *coupon rate* on newly issued five-year Singapore government bonds is 2.5%. For an investor seeking a low-risk investment maturing in five years, do the bonds issued twenty-five years ago with a much higher *coupon rate* provide a more attractive return than the new five-year bonds? What about the ten-year bonds issued five years ago?

A4-6. The YTM on each of these bonds would be quite similar. The bonds paying an 8% coupon would trade at a significant premium, and the bonds paying a 5% coupon would trade at a smaller premium. The premium that an investor would have to pay to acquire either of these bonds would largely offset the higher coupon rates that they offer, leaving the YTM close to 2.5%.

Q4-7. Describe how and why a bond's *interest rate risk* is related to its maturity.

A4-7. Generally speaking, interest rate risk increases with time to maturity because more maturity allows more potential changes in the interest rate to occur.

Q4-8. Explain why the *yield-to-maturity* on a *junk bond* is not a particularly good measure of the return you can expect if you buy it and hold it until maturity.

A4-8. Junk bonds represent a significant probability of default. The YTM is the return that an investor earns if they hold a bond to maturity and all cash payments are made in full and on time. Investors holding junks bonds know that default sometimes occurs, so on average they do not expect all payments to be made. Therefore, the expected return on junk bonds is less than the YTM.

Q4-9. Under the *expectations* theory, what does the slope of the *yield curve* reveal about the future path of interest rates?

A4-9. Under the *expectations hypothesis*, the slope of the yield curve indicates the direction of future interest rate movements. A positive slope indicates rising rates, and a negative slope indicates falling rates.

Q4-10. If the yield curve typically slopes upward, what does this imply about the long-term path of interest rates if the expectations theory is true?

A4-10. This would imply that investors normally expect rates to rise.

Solutions to End-of-Chapter Problems

Valuation Basics

P4-1. A best-selling author decides to cash in on her latest novel by selling the rights to the book's royalties for the next six years to an investor. Royalty payments arrive once per year, starting one year from now. In the first year the author expects $400,000 in royalties, followed by $300,000, then $100,000, and then $10,000 in the three subsequent years. If the investor purchasing the rights to royalties requires a return of 7% per year, what should the investor pay?

A4-1. $$P = \frac{\$400{,}000}{1.07^1} + \frac{\$300{,}000}{1.07^2} + \frac{\$100{,}000}{1.07^3} + \frac{\$10{,}000}{1.07^4} + \frac{\$10{,}000}{1.07^5} + \frac{\$10{,}000}{1.07^6} = \$738{,}915.42$$

P4-2. An oil well produces 20,000 barrels of oil per year. Suppose the price of oil is $50 per barrel. You want to purchase the right to the oil produced by this well for the next five years. At a discount rate of 10%, what is the value of the oil rights? (You can assume that the cash flows from selling oil arrive at annual intervals).

A4-2. $$\mathrm{P} = \frac{\$1{,}000{,}000}{1.10^1} + \frac{\$1{,}000{,}000}{1.10^2} + \frac{\$1{,}000{,}000}{1.10^3} + \frac{\$1{,}000{,}000}{1.10^4} + \frac{\$1{,}000{,}000}{1.10^5} = \$3{,}790{,}787$$

Bond Prices and Interest Rates

P4-3. A $1,000 par value bond makes two interest payments each year of $45 each. What is the bond's *coupon rate*?

A4-3. 9% = 45(2)/1,000

P4-4. A $1,000 par value bond has a *coupon rate* of 8 % and a *coupon yield* of 9 %. What is the bond's market price?

A4-4. The annual coupon is $80. To find the price, solve 0.09 = $80/P, P = $888.89

P4-5. A bond sells for $900 and offers a *coupon yield* of 7.2 %. What is the bond's annual coupon payment?

A4-5. $X/$900 = 0.072, so X = $64.80

P4-6. A bond offers a *coupon rate* of 5 %. If the par value is $1,000 and the bond sells for $1,250, what is the *coupon yield*?

A4-6. $50/$1,250 = 0.04, or 4%.

P4-7. A bond makes two $45 interest payments each year. Given that the bond's par value is $1,000 and its price is $1,050, calculate the bond's *coupon rate* and *coupon yield*.

A4-7. Coupon yield is $90/$1,050 = 0.0857 or 8.57%. Coupon rate = $90/$1,000 = 0.09, or 9%.

P4-8. Calculate the price of a 5-year, $1,000 par value bond that makes semiannual payments, has a *coupon rate* of 8 %, and offers a *yield to maturity* of 7 %. Recalculate the price assuming a 9 % YTM. What is the general relationship that this problem illustrates?

A4-8.
$$P = \frac{40}{(1+\frac{0.07}{2})} + \frac{40}{(1+\frac{0.07}{2})^2} + \frac{40}{(1+\frac{0.07}{2})^3} + ... + \frac{1{,}040}{(1+\frac{0.07}{2})^{10}} = 1{,}041.58$$

$$P = \frac{40}{(1+\frac{0.09}{2})} + \frac{40}{(1+\frac{0.09}{2})^2} + \frac{40}{(1+\frac{0.09}{2})^3} + ... + \frac{1{,}040}{(1+\frac{0.09}{2})^{10}} = 960.44$$

When the *coupon rate* is higher than the *YTM*, the bond sells at a *premium*, but when the coupon rate is lower than the YTM, the bond sells at a discount. This problem also illustrates the general inverse relationship between bond prices and interest rates.

P4-9. A $1,000 par value bond makes annual interest payment of $75. If it offers a *yield to maturity* of 7.5 %, what is the price of the bond?

A4-9. The bond will sell at par value or $1,000 because the bond's coupon rate (75/1000 = 7.5%) is equal to its YTM. The idea here is that a bond's market price equals par value if *yield to maturity* is equal to the bond's coupon rate.

P4-10. A $1,000 par value bond pays a *coupon rate* of 8.2 %. The bond makes semiannual payments, and it matures in four years. If investors require a 10 % return on this investment, what is the bond's price?

A4-10.
$$P = \frac{41}{1.05} + \frac{41}{1.05^2} + \frac{41}{1.05^3} + ... + \frac{1{,}041}{1.05^8} = \$941.83$$

P4-11. Griswold Travel has issued 6-year bonds that pay $30 in interest twice each year. The par value of these bonds is $1,000 and they offer a *yield to maturity* of 5.5 %. How much are the bonds worth?

A4-11.
$$P = \frac{30}{(1+\frac{0.055}{2})} + \frac{30}{(1+\frac{0.055}{2})^2} + ... \frac{1{,}030}{(1+\frac{0.055}{2})^{12}} = 1{,}025.26$$

P4-12. Bennifer Jewellers recently issued 10-year bonds that make annual interest payments of $50. Suppose you purchased one of these bonds at par value when it was issued, and right away market interest rates jumped and the YTM on your bond rose to 6 %. What happened to the price of your bond?

A4-12 $P = \frac{50}{1.06} + \frac{50}{1.06^2} + \frac{50}{1.06^3} + ... + \frac{1{,}050}{1.06^{10}} = \$926.40$ so the price fell by $73.60.

P4-13. You are evaluating two similar bonds. Both mature in 4 years, both have a $1,000 *par value*, and both pay a *coupon rate* of 10 %. However, one bond pays that coupon in annual instalments, whereas the other makes semiannual payments. Suppose you require a 10 % return on either bond. Should these bonds sell at identical prices or should one be worth more than the other? Use Equations 4.2a and 4.3a and let *r* = 10%. What prices do you obtain for these bonds? Can you explain the apparent paradox?

A4-13. Using Equation 4.2a, the bond that pays annual interest will sell at par value, and using Equation 4.3a, the semiannual bond will also sell for par value. Intuitively, the bond that makes semiannual payments should be worth more because it pays the same cash as the other bond but it pays the cash a little sooner. The fact that Equations 4.2a and 4.3a make it seem as if the two bonds will both sell at par is a result of the fact that a 10% annual discount rate is not equal to a 5% semiannual discount rate. That is, when we let *r* = 0.10 in Equation 4.3a, we are really using an annual discount rate of 10.25%. It is only because we are applying a higher effective discount rate to the semiannual bond that it appears to be just as valuable as the bond that pays annually.

P4-14. A bond makes annual interest payments of $75. The bond matures in 4 years, has a *par value* of $1,000, and sells for $975.30. What is the bond's *yield to maturity* (YTM)?

A4-14. $975.30 = \frac{75}{1+r} + \frac{75}{(1+r)^2} + \frac{75}{(1+r)^3} + \frac{1{,}075}{(1+r)^4}$ so r = 0.0825 or 8.25%

P4-15. Johanson VI Advisers issued $1,000 par value bonds a few years ago with a coupon rate of 7 %, paid semiannually. After the bonds were issued, interest rates fell. Now with three years remaining before they mature, the bonds sell for $1,055.08. What YTM do these bonds offer?

A4-15. $1{,}055.08 = \frac{35}{1+\frac{r}{2}} + \frac{35}{(1+\frac{r}{2})^2} + \frac{35}{(1+\frac{r}{2})^3} + ... + \frac{1{,}035}{(1+\frac{r}{2})^6}$ and r =0.05 or 5%

P4-16. A bond offers a 6 % coupon rate and sells at par. What is the bond's yield to maturity?

A4-16. 6%

P4-17. You have gathered the following data on three bonds:

Bond	Maturity	Coupon %
A	10 yrs	9%
B	9 yrs	1%
C	5 yrs	5%

a. If the market's required return on all three bonds is 6%, what are the market prices of the bonds (you can assume annual interest payments).
b. The market's required return suddenly rises to 7%. What are the new bonds' prices, and what is the percentage change in price for each bond?
c. If the market's required return falls from the initial 6% to 5%, what are the new prices, and what is the percentage change in each price relative to the answer obtained in part (a)?
d. Which bond's price is most sensitive to interest rate movements? Does this answer surprise you? Why or why not? Can you explain why this bond's price is so sensitive to rate changes?
e. Which bond's price is least sensitive to interest rate movements? Explain.

A4-17.

a. Bond A = $1,220.80
Bond B = $659.92
Bond C = $957.88

b. Bond A = $1,140.47 a decrease of about 6.6%
Bond B = $609.09 a decrease of about 7.7%
Bond C = $918.00 a decrease of about 4.2%

c. Bond A = $1,308.87 an increase of 7.2%
Bond B = $715.69 an increase of 8.4%
Bond C = $1,000 an increase of 4.4%

d. Bond B's price is the most sensitive to rate changes. This may be a little surprising because Bond A has a longer maturity than Bond B, but Bond A also has a much higher coupon. Because Bond A's coupon is so much higher and it pays quite a bit of cash flow in the early years of its life, its price is not as sensitive to rate changes as is Bond B's price.

e. Bond C's price is the least sensitive to rate changes, which is not surprising because its maturity is so much shorter than the other two bonds.

P4-18. The rate of inflation is 5 % and the *real interest rate* is 3 %. What is the *nominal interest rate*?

A4-18. 1.05 x 1.03 – 1 = 0.0815 or 8.15%. A rough approximation would be 5% + 3% = 8%.

P4.19. The nominal interest rate is 9 % and the inflation rate is 7 %. What is the *real interest rate*?

A4-19. 1.09/1.07 – 1 = 0.0187 or 1.87%. A rough approximation would be 9% − 7% = 2% real rate.

Types of Bonds

P4-20. Suppose investors face a tax rate of 40 % on interest received from corporate bonds. Suppose AAA-rated corporate bonds currently offer yields of about 7 %. Approximately what yield would AAA-rated tax-free bonds need to offer to be competitive?

A4-20. The after-tax return on the corporate bonds would be 7%(1 − 40%) = 4.2%. Since tax-free bond interest is tax free, municipal bonds could offer a 4.2% yield and be competitive with corporate bonds.

P4-21. Investors face a tax rate of 33% on interest paid by corporate bonds. If tax-free bonds currently offer yields of 6 %, what yield would equally risky corporate bonds need to offer to be competitive?

A4-21. 0.06 / (1 – 0.33) = 0.06/0.67 = 0.0896, or 8.96%.

P4-22. You purchase an Australian Government inflation-indexed bond at *par value* of $1,000. The bond offers a coupon rate of 6 % paid semiannually. During the first six months that you hold the bond, prices in Australia rise by 2%. What is the new par value of the bond, and what is the amount of your first coupon payment?

A4-22. Par value rises to $1,020 and the coupon equals ½ ($1,020)(0.06) = $30.60.

P4-23. What is the price of a zero-coupon bond that has a par value of $1,000? The bond matures in thirty years and offers a yield to maturity of 4.5 %. Calculate the price one year later when the

bond has 29 years left before it matures (assume the yield remains at 4.5%). What is the return that investors earn if they buy the bond with thirty years remaining and sell it one year later?

A4-23. $1,000/(1.045)^{30} = \$267.00$. $1,000/(1.045)^{29} = \$279.02$. The one-year return is therefore (279.02/267) –1 = 0.045, or 4.5%. This is just the required return on the bond.

P4.24. A zero-coupon bond has a $1,000 face value, matures in 10 years, and currently sells for $781.20.
a. What is the market's required return on this bond?
b. Suppose you hold this bond for 1 year and sell it. At the time you sell the bond, market rates have increased to 3.5%. What return did you earn on this bond?
c. Suppose that rather than buying the 10-year zero-coupon bond described at the start of this problem, you instead purchased a 10-year 2.5% coupon bond (assume annual payments). Because the bond's coupon rate equalled the market's required return at the time of purchase, you paid par value ($1,000) to acquire the bond. Again assume that you held the bond for one year, received one coupon payment, and then sold the bond, but at the time of sale the market's required return was 3.5%. What was your return for the year? Compare your answer here to your answer in part (b).

A4.24.

a. To find the required return, or the YTM, solve this equation for r:
$781.20 = $1,000/(1+r)^10
r = 2.5%
b. One year from now the bond will have 9 years remaining before maturity. If the market's required return at that time is 3.5%, the bond's price will be
$1,000/(1.035)^9 = $733.73
This means you lost $47.47 during the year ($781.20 – $733.73) for a percentage return of –6.08% (–$47.47/$781.20).
c. At the end of the year, the bond would have 9 years remaining. To calculate its price, just use Excel and plug in 3.5% for the rate, 9 years for the number of period, –25 for the coupon payment, and –1000 for the principal payment at the end. In Excel you type =pv(0.035,9,-25,-1000,0) = $923.92.

To calculate your return for the year, recognise that you lost $76.08 on the sale of the bond ($1,000 – $923.92), but partially offsetting that was the $25 coupon payment that you received. So your total loss was $51.08 ($76.08 – $25), or –5.108% on a percentage basis (–$51.08/$1,000).

To summarise, whether you purchased the zero-coupon or the 2.5% coupon bond, you lost money due to an increase in interest rates. Your loss was greater (on a percentage basis) on the zero-coupon bond. In other words, the zero-coupon bond has greater interest rate risk than the 2.5% coupon bond, even though both bonds have the same maturity.

Bond Markets

P4-25. A corporate bond's value index is quoted as 98.110. What is the price of the bond if its par value is $1,000?

A4-25. 98.110% of par value, or $981.10.

P4-26. A corporate bond's value index is quoted as 102.312. If the bond's par value is $1,000, what is its market price?

A4-26. $1,023.12

P4.27. A corporate bond's value index is listed at 102.801. It matures in 3 years, has a coupon rate of 5%, and pays interest semiannually. What is the bond's yield to maturity?

A4.27. First notice that the bond is selling at a premium, so its YTM must be less than 5%, the coupon rate. Either through trial and error or by using a calculator or Excel, we can determine that the YTM is 4% per year.

The Term Structure of Interest Rates

P4-28. A one-year government security offers a 4 % *yield to maturity* (YTM). A two-year government security offers a 4.25 % YTM. According to the *expectations theory*, what is the expected interest rate on a one-year security next year?

A4-28. $(1.0425)^2 = (1.04)(1+r)$, so r = 0.045, or 4.5%

P4-30. A one-year short-term government bond offers a 6 % *yield to maturity*. The market's consensus forecast is that one-year government bonds will offer 6.25 % next year. What is the current yield on a 2-year government bond if the *expectations theory* holds?

A4-30. $(1+r)^2 = (1.06)(1.0625)$, so r = 0.06125, or 6.125%

Answer to MiniCase

Valuing Bonds

As you browse opportunities for investment, you come across Horsham Industries, which has some bonds on issue. Based on this online information, answer the following questions.

Issuer name	Coupon	Maturity	Rating Moody's	High	Low	Last	Change	Yield %
Horsham Industries	5.550%	Mar 2037	Aa3	96.124	95.933	96.124	–0.245	5.825

Assignment

1. What is the YTM for this Horsham Industries company bond?
2. What is the coupon yield of this bond over the next year?
3. If your required rate of return for a bond of this risk-class is 6.2 %, what value do you place on this Horsham Industries bond?
4. At the required rate of return of 6.2 % are you interested in purchasing this bond?
5. If you purchased this Horsham Industries bond for $961.24 yesterday and the market rate of interest for this bond increased to 6.0 % today, do you have a gain or loss? How much is that gain or loss in dollars?

Answers

1. n = 29 Years $\times$ 2 = 58
   PV = $961.24
   PMT = $55.50/2 = $27.75
   FV = 1,000
   i = YTM = 2.91427% $\times$ 2
   YTM = 5.8285%

2. Coupon yield = Annual coupon/Price
   = $55.50/$961.24
   = 5.77%

3. n = 58
   i = 6.2/2 = 3.1%
   PMT = $27.75
   FV = $1,000
   PV = $913.00

4. No, cost is $961.24 but you value it at $913.

5. Rates increase to 6%
   Price drops to $938.50. That represents a loss of $22.73.